

Use of Non-Orthogonal Multiple Access in Dual-hop relaying

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Abstract

To improve the sum-rate (SR) of the dual-hop relay system, a novel two-stage power allocation scheme with non-orthogonal multiple access (NOMA) is proposed. In this scheme, after the reception of the superposition coded symbol with a power allocation from the source, the relay node forwards a new superposition coded symbol with another power allocation to the destination. By employing the maximum ratio combination (MRC), the destination jointly decodes the information symbols from the source and the relay. Assuming Rayleigh fading channels, closed-form solution of the ergodic SR at high signal-to-noise ratio (SNR) is derived and a practical power allocation is also designed for the proposed NOMA scheme. Through numerical results, it is shown that the performance of the proposed scheme is significantly improved compared with the existing work.

Index Terms: Non-orthogonal multiple access (NOMA), cooperative relay system (CRS), sum-rate (SR).

I. INTRODUCTION

Recently, non-orthogonal multiple access (NOMA) technique is widely considered as a promising multiple access (MA) candidate for 5G mobile networks due to its superior spectral efficiency [1], [2]. The key idea of NOMA is to explore the power domain for realizing MA, where different users are served at different power levels [3] and [4]. Consequently, the impact of different choices of power allocation coefficients has been studied in [5], where the authors considered the fairness of the user for NOMA systems. Under the total transmit power constraint and the minimum rate constraint of the weak user, the ergodic capacity maximization problem for multiple-input multiple-output (MIMO) NOMA systems has been solved [6].

In addition, the implementation of NOMA with two base stations (BSs) has been investigated in [7]. The authors derived the sum rates (SRs) for NOMA schemes with superposition coding (SC) that does not require instantaneous channel state information (CSI) at BSs. On this basis, the work of NOMA in the coordinated direct and relay transmission (CDRT) has been introduced in [4]. The authors proposed the cooperative relaying system (CRS) using NOMA, and presented the exact and asymptotic expressions for the achievable rate of the proposed system in independent Rayleigh fading channels. Nevertheless, the achievable rate is limited by the source-to-destination link which leading to the loss of the system performance.

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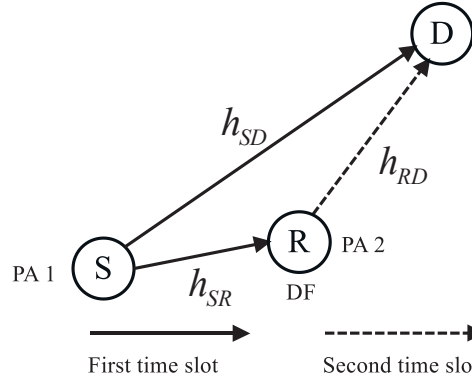


Fig. 1. The Dual-hop relay system with two-stage power allocation.

To solve this problem, we propose a two-stage power allocation CRS using NOMA in this paper. In our proposed scheme, once MRC at the destination and twice power allocations at the source and the relay are employed. To further improve the performance of the ergodic SR of our proposed scheme and the existing work [4], the destination will not immediately decode the reception from the source until it receives the superposition coded signal from the relay. We focus on the analysis of the achievable ergodic SR for our proposed scheme, and the closed-form expression at SNR region is then derived. In addition, to maximize the ergodic SR, a practical two-stage power allocation strategy is designed. Numerical results are presented to corroborate the derived theoretical analysis, and show that the proposed scheme significantly improves the ergodic SR compared to the one in [4].

II. SYSTEM MODEL AND PROPOSED SCHEME

A simple CRS consisting of one source, one relay, and one destination is shown in Fig. 1. We assume all nodes operate in half-duplex mode and the direct link between the source and the destination exists. The channels from the source to the destination, from the source to the relay, and from the relay to the destination are denoted as h_{SD} , h_{SR} , and h_{RD} , respectively, which are assumed to be independent complex Gaussian random variables with variances α_{SD} , α_{SR} , and α_{RD} , respectively. In our proposed scheme, each transmission involves two time slots. At the first time slot, assuming the adoption of the superposition code, the signal $\sqrt{a_1 P_t} x_1 + \sqrt{a_2 P_t} x_2$ is simultaneously transmitted from the source to the relay and the destination, where x_i , $i=1,2$, denote the broadcasted symbols at the source, a_1 and a_2 with $a_1 + a_2 = 1$ are the power allocation factors, and P_t stands for the total transmit power. The received signal at the source and the relay are given by

$$y_R = h_{SR} \left(\sqrt{a_1 P_t} x_1 + \sqrt{a_2 P_t} x_2 \right) + n_R, \quad (1)$$

$$y_D^{(1)} = h_{SD} \left(\sqrt{a_1 P_t} x_1 + \sqrt{a_2 P_t} x_2 \right) + n_D^{(1)}, \quad (2)$$

where $\{n_R, n_D^{(1)}\} \sim CN(0, \sigma^2)$ represent the additive white Gaussian noises (AWGNs) with zero mean and variance σ^2 .

To successfully and simultaneously decode x_1 and x_2 at the relay, we further assume that the path loss and shadowing effects for h_{SD} are worse than those for h_{SR} , which leading the ordinary channel condition as $\alpha_{SD} < \alpha_{SR}$. By treating x_2 as noise to decode x_1 , and then using the SIC to acquire x_2 , the relay can efficiently decode x_1 and x_2 from (1) during one time slot. By this way, the SNRs for x_1 and x_2 at the relay can be respectively expressed as

$$\gamma_R^{(x_1)} = \frac{|h_{SR}|^2 a_1 P_t}{|h_{SR}|^2 a_2 P_t + \sigma^2}, \quad \gamma_R^{(x_2)} = \frac{|h_{SR}|^2 a_2 P_t}{\sigma^2}. \quad (3)$$

Note that different from [4], to improve the system performance, the destination will not decode the received signal from the source but instead conserves it until the second time slot comes.

At the second time slot, the relay node forwards a new symbol x_R with SC to the destination:

$$x_R = \sqrt{a_3 P_t} x_1 - \sqrt{a_4 P_t} x_2, \quad (4)$$

where P_t is the total transmit power, a_3 and a_4 with $a_3 + a_4 = 1$ are new power allocation coefficients. Therefore, the received signal at the destination can be written as

$$y_D^{(2)} = h_{RD} \left(\sqrt{a_3 P_t} x_1 - \sqrt{a_4 P_t} x_2 \right) + n_D^{(2)}, \quad (5)$$

where $n_D^{(2)}$ is the AWGN at the destination with zero mean and variance σ^2 . From above, we see that there are two signals received at the destination, which are $y_D^{(1)}$ and $y_D^{(2)}$. To jointly decode x_1 and x_2 at the destination, we employ $y_D^{(1)} \sqrt{a_4} h_{R,D} + y_D^{(2)} \sqrt{a_2} h_{S,D}$ and $y_D^{(1)} \sqrt{a_4} h_{R,D} - y_D^{(2)} \sqrt{a_2} h_{S,D}$. Therefore, the target signals can be expressed as

$$T_{x_1} = h_{SD} h_{RD} \varsigma \sqrt{P_t} x_1 + \sqrt{a_4} h_{RD} n_D^{(1)} + \sqrt{a_2} h_{SD} n_D^{(2)}, \quad (6)$$

$$T_{x_2} = h_{SD} h_{RD} \varsigma \sqrt{P_t} x_2 + \sqrt{a_3} h_{RD} n_D^{(1)} + \sqrt{a_1} h_{SD} n_D^{(2)}, \quad (7)$$

where $\varsigma = \sqrt{a_1 a_4} + \sqrt{a_2 a_3}$. According to (6) and (7), the corresponding receive SNR for x_1 and x_2 can be obtained as

$$\gamma_D^{(x_1)} = \frac{|h_{SD}|^2 |h_{RD}|^2 \varsigma^2 \rho}{a_4 |h_{RD}|^2 + a_2 |h_{SD}|^2}, \quad (8)$$

and

$$\gamma_D^{(x_2)} = \frac{|h_{SD}|^2 |h_{RD}|^2 \varsigma^2 \rho}{a_3 |h_{RD}|^2 + a_1 |h_{SD}|^2}, \quad (9)$$

respectively, where $\rho = \frac{P_t}{\sigma^2}$ denotes the transmit SNR.

III. ACHIEVABLE SR AND PERFORMANCE ANALYSIS

In this section, in order to characterize the superiority of our proposed scheme, we will focus on the analysis the achievable ergodic SR and derive its closed-form expression at high SNR.

The achievable rate associated with symbol x_1 is obtained using (3) and (8) as

$$\begin{aligned} C_{x_1} &= \min \left\{ \frac{1}{2} \log \left(1 + \gamma_D^{(x_1)} \right), \frac{1}{2} \log \left(1 + \gamma_R^{(x_1)} \right) \right\} \\ &= \frac{1}{2} \log \left(1 + \min \left\{ \gamma_D^{(x_1)}, \gamma_R^{(x_1)} \right\} \right), \end{aligned} \quad (10)$$

where $1/2$ is resulted from the dual-hop transmission in two time slots. Similarly, according to (3) and (9), the achievable rate associated with symbol x_2 is obtained as

$$\begin{aligned} C_{x_2} &= \min \left\{ \frac{1}{2} \log \left(1 + \gamma_D^{(x_2)} \right), \frac{1}{2} \log \left(1 + \gamma_R^{(x_2)} \right) \right\} \\ &= \frac{1}{2} \log \left(1 + \min \left\{ \gamma_D^{(x_2)}, \gamma_R^{(x_2)} \right\} \right) \\ &= \frac{1}{2} \log \left(1 + \min \left\{ \tilde{\gamma}_D^{(x_2)}, \tilde{\gamma}_R^{(x_2)} \right\} \rho \right), \end{aligned} \quad (11)$$

where $\tilde{\gamma}_D^{(x_i)} = \frac{1}{\rho} \gamma_D^{(x_i)}$, for $i = 1, 2$. Synthesizing (10) and (11), the achievable SR can be expressed as

$$C_{sum} = C_{x_1} + C_{x_2}. \quad (12)$$

Further denoting $|h_{SR}|^2 = \beta_{SR}$, $|h_{RD}|^2 = \beta_{RD}$, and $|h_{SD}|^2 = \beta_{SD}$, we have

$$\min \{ \gamma_D^{(x_1)}, \gamma_R^{(x_1)} \} = \min \left\{ \frac{\zeta^2 \rho \beta_{SD} \beta_{RD}}{a_2 \beta_{SD} + a_4 \beta_{RD}}, \frac{\beta_{SR} a_1 \rho}{\beta_{SR} a_2 \rho + 1} \right\},$$

and

$$\min \{ \tilde{\gamma}_D^{(x_2)}, \tilde{\gamma}_R^{(x_2)} \} = \min \left\{ \frac{\zeta^2 \beta_{SD} \beta_{RD}}{a_1 \beta_{SD} + a_3 \beta_{RD}}, a_2 \beta_{SR} \right\}.$$

Letting $X = \min \{ \gamma_D^{(x_1)}, \gamma_R^{(x_1)} \}$, the complementary cumulative distribution function (CCDF) of X can be obtained as

$$\overline{F}_X(x) = Pr \left\{ \frac{m \rho \beta_{SD} \beta_{RD}}{a_2 \beta_{SD} + a_4 \beta_{RD}} > x, \frac{\beta_{SR} a_1 \rho}{\beta_{SR} a_2 \rho + 1} > x \right\}. \quad (13)$$

Noting that the CCDF of $\beta_\delta = e^{-\frac{x}{\alpha_\delta}}$, for $\delta \in \{SR, SD, RD\}$, (13) can be equivalently represented as

$$\begin{aligned}\bar{F}_X(x) &= \bar{F}_{SR}\left(\frac{x}{a_1\rho - a_2\rho x}\right) \left[Pr\left\{\beta_{SD} > \frac{a_4\beta_{RD}x}{\varsigma^2\rho\beta_{RD} - a_2x} \middle| \beta_{RD} > \frac{a_2x}{\varsigma^2\rho}\right\} \bar{F}_{RD}\left(\frac{a_2x}{\varsigma^2\rho}\right) \right. \\ &\quad \left. + Pr\left\{\beta_{SD} < \frac{a_4\beta_{RD}x}{\varsigma^2\rho\beta_{RD} - a_2x} \middle| \beta_{RD} < \frac{a_2x}{\varsigma^2\rho}\right\} F_{RD}\left(\frac{a_2x}{\varsigma^2\rho}\right) \right] \\ &= \frac{1}{\alpha_{RD}} e^{-\frac{x}{(a_1\rho - a_2\rho x)\alpha_{SR}} - \frac{a_2x}{\varsigma^2\rho\alpha_{RD}}} \int_{\frac{a_2x}{\varsigma^2\rho}}^{\infty} e^{-\frac{a_4u}{(\varsigma^2\rho u - a_2x)\alpha_{SD}} - \frac{u}{\alpha_{RD}}} du \\ &\quad + \frac{1}{\alpha_{RD}} e^{-\frac{x}{(a_1\rho - a_2\rho x)\alpha_{SR}}} \times \left(1 - e^{-\frac{a_2x}{\varsigma^2\rho\alpha_{RD}}}\right) \int_0^{\frac{a_2x}{\varsigma^2\rho}} e^{-\frac{a_4u}{(\varsigma^2\rho u - a_2x)\alpha_{SD}} - \frac{u}{\alpha_{RD}}} du.\end{aligned}\quad (14)$$

Consider the high transmit SNR case, i.e., $\rho \gg 1$. In this case, we have

$$\frac{\beta_{SR}a_1\rho}{\beta_{SR}a_2\rho + 1} \sim \frac{a_1}{a_2}.$$

Letting $t_1 = \varsigma^2\rho u - a_2x$ and using $\int_0^\infty e^{-\frac{a}{x} - bx} dx = 2\sqrt{\frac{a}{b}} K_1\left(2\sqrt{ab}\right)$ [9, 3.324.1], when $x < \frac{a_1}{a_2}$, (14) can be equivalently written as

$$\begin{aligned}\bar{F}_X(x) &= \frac{e^{-\frac{a_2x}{\varsigma^2\rho\alpha_{RD}} - \frac{a_4x}{\varsigma^2\rho\alpha_{SD}}}}{\varsigma^2\rho\alpha_{RD}} \int_0^\infty e^{-\frac{a_2a_4x^2}{\varsigma^2\rho t_1\alpha_{SD}} - \frac{t_1}{\varsigma^2\rho\alpha_{RD}}} dt_1 \\ &= \frac{2x}{\varsigma^2\rho\xi\phi_1\alpha_{RD}} e^{-\frac{a_2x}{\varsigma^2\rho\alpha_{RD}} - \frac{a_4x}{\varsigma^2\rho\alpha_{SD}}} K_1\left(\frac{2\xi x}{\rho\phi_1}\right),\end{aligned}\quad (16)$$

where $\xi = \sqrt{\frac{1}{\varsigma^2\alpha_{RD}}}$, $\phi_1 = \sqrt{\frac{\varsigma^2\alpha_{SD}}{a_2a_4}}$, and $K_1(\cdot)$, denotes the first order modified Bessel function of the second kind [8]. For

the case $x > \frac{a_1}{a_2}$, $\bar{F}_X(x) = 0$ always holds true due to $\frac{\beta_{SR}a_1\rho}{\beta_{SR}a_2\rho + 1} < \frac{a_1}{a_2}$. The achievable rate can be calculated as

$$\begin{aligned}\tilde{C}_{x_1} &= \int_0^{\frac{a_1}{a_2}} \frac{1}{2} \log_2(1+x) dF_X(x) + \frac{1}{2} \log_2\left(1 + \frac{a_1}{a_2}\right) \left(1 - F_X\left(\frac{a_1}{a_2}\right)\right) \\ &= \frac{1}{2} \log_2\left(1 + \frac{a_1}{a_2}\right) - \frac{1}{2\ln 2} \int_0^{\frac{a_1}{a_2}} \frac{1}{1+x} \left(1 - \frac{1}{m\rho\xi\phi_x\alpha_{RD}} 2xe^{-\frac{a_2x}{m\rho\alpha_{RD}} - \frac{a_4x}{m\rho\alpha_{SD}}} K_1\left(\frac{2\xi x}{\rho\phi_x}\right)\right) dx,\end{aligned}\quad (17)$$

where the first equality holds due to $\int_0^\infty \frac{1}{2} \log_2(1+x) f_X(x) dx = \frac{1}{2\ln 2} \int_0^\infty \frac{1-F_X(x)}{1+x} dx$ with $F_X(x) = 1 - \bar{F}_X(x)$. For small

x , $K_\nu(x) \approx \frac{\Gamma(\nu)}{2} \left(\frac{2}{x}\right)^\nu$ [10], where $\Gamma(\cdot)$ denotes the gamma function. Therefore, (17) can be approximately rewritten as

$$\begin{aligned}\tilde{C}_{x_1} &= \frac{1}{2} \log_2\left(1 + \frac{a_1}{a_2}\right) - \frac{1}{2\ln 2} \int_0^{\frac{a_1}{a_2}} \frac{1 - e^{-\frac{a_2x}{\varsigma^2\rho\alpha_{RD}} - \frac{a_4x}{\varsigma^2\rho\alpha_{SD}}}}{1+x} dx \\ &= \frac{e^{\frac{1}{\varsigma^2\rho}\left(\frac{a_2}{\alpha_{RD}} + \frac{a_4}{\alpha_{SD}}\right)}}{2\ln 2} \left[\text{Ei}\left(-\frac{a_1}{a_2\varsigma^2\rho}\left(\frac{a_2}{\alpha_{RD}} + \frac{a_4}{\alpha_{SD}}\right)\right) - \text{Ei}\left(-\frac{1}{\varsigma^2\rho}\left(\frac{a_2}{\alpha_{RD}} + \frac{a_4}{\alpha_{SD}}\right)\right) \right],\end{aligned}\quad (18)$$

where $\text{Ei}(\cdot)$ denotes the exponential integral function, and the integral result $\int_0^u \frac{e^{-\mu x} dx}{x+\beta} = e^{\mu\beta} [\text{Ei}(-\mu u - \mu\beta) - \text{Ei}(-\mu\beta)]$

[9, 3.352.1] is used in the first equality. Considering high SNR case, (13) can be equivalently rewritten as

$$\min\{\gamma_D^{(x_1)}, \gamma_R^{(x_1)}\} = \min\left\{\rho \times \frac{m\beta_{SD}\beta_{RD}}{a_2\beta_{SD} + a_4\beta_{RD}}, \frac{a_1}{a_2}\right\} \triangleq \frac{a_1}{a_2}, \quad (19)$$

which leading $C_{x_1} \triangleq \frac{1}{2}\log_2\left(1 + \frac{a_1}{a_2}\right)$. Also, for (18), we have $\frac{1}{\zeta^2\rho}\left(\frac{a_2}{\alpha_{RD}} + \frac{a_4}{\alpha_{SD}}\right) \approx 0$ with $\rho \gg 1$ and the achievable rate $\tilde{C}_{x_1} \triangleq \frac{1}{2}\log_2\left(1 + \frac{a_1}{a_2}\right)$. It is clear that there is an good match between the analytical result in (18) and (13). Similarly, letting $Y = \min\{\tilde{\gamma}_D^{(x_2)}, \tilde{\gamma}_R^{(x_2)}\}$ the CCDF of Y can be obtained as

$$\overline{F}_Y(y) = \overline{F}_Y^{(1)}(y) \times \overline{F}_Y^{(2)}(y), \quad (20)$$

where $\overline{F}_Y^{(1)}(y) = Pr\left\{\frac{\zeta^2\beta_{SD}\beta_{RD}}{a_1\beta_{SD} + a_3\beta_{RD}} > y\right\}$ and $\overline{F}_Y^{(2)}(y) = Pr\{a_2\beta_{SR} > y\}$. Since $\overline{F}_Y^{(1)}(y)$ can be rewritten as

$$\overline{F}_Y^{(1)}(y) = Pr\left\{\frac{\zeta^2}{2a_1a_3} \cdot \frac{2a_1a_3\beta_{SD}\beta_{RD}}{a_1\beta_{SD} + a_3\beta_{RD}} > y\right\}, \quad (21)$$

using [11, Theorem 1], we have

$$\overline{F}_Y^{(1)}(y) = \frac{2y}{\zeta^2} \sqrt{\frac{a_1a_3}{\alpha_{SD}\alpha_{RD}}} e^{-\frac{(a_1\alpha_{SD} + a_3\alpha_{RD})y}{\zeta^2\alpha_{SD}\alpha_{RD}}} K_1\left(\frac{2y}{\zeta^2} \sqrt{\frac{a_1a_3}{\alpha_{SD}\alpha_{RD}}}\right). \quad (22)$$

Therefore, the CCDF of Y can be finally expressed as

$$\overline{F}_Y(y) = \frac{2\gamma y}{\rho} e^{-\left(\frac{a_1\alpha_{SD} + a_3\alpha_{RD}}{\zeta^2\alpha_{SD}\alpha_{RD}} + \frac{1}{a_2\alpha_{SR}}\right)\frac{y}{\rho}} K_1\left(\frac{2\gamma y}{\rho}\right), \quad (23)$$

where $\gamma = \frac{1}{\zeta^2} \sqrt{\frac{a_1a_3}{\alpha_{SD}\alpha_{RD}}}$. As a double check, the CDF $F_Y(y) = 1 - \overline{F}_Y(y)$ in (23) was validated by Monte Carlo simulation, as illustrated in Fig. 2. It is clear that there is an excellent match between the analytical result in (23) and the Monte Carlo simulation.

Assuming $\left(\frac{a_1\alpha_{SD} + a_3\alpha_{RD}}{\zeta^2\alpha_{SD}\alpha_{RD}} + \frac{1}{a_2\alpha_{SR}}\right)\frac{1}{\rho} = \eta$, and using [9, 3.352.2], the achievable rate of x_2 can be obtained as

$$\tilde{C}_{x_2} = \frac{\gamma}{\rho \ln 2} \int_0^\infty \frac{ye^{-\eta y} K_1\left(\frac{2\gamma y}{\rho}\right)}{1 + y} dy = -\frac{e^\eta}{2 \ln 2} \text{Ei}(-\eta). \quad (24)$$

Finally, putting (18) and (24) together, we can express the ergodic SR of our proposed system in closed form as

$$\hat{C}_{sum} = \frac{e^{\frac{1}{\zeta^2\rho}\left(\frac{a_2}{\alpha_{RD}} + \frac{a_4}{\alpha_{SD}}\right)}}{2 \ln 2} \left[\text{Ei}\left(-\frac{a_1}{a_2\zeta^2\rho}\left(\frac{a_2}{\alpha_{RD}} + \frac{a_4}{\alpha_{SD}}\right)\right) - \text{Ei}\left(-\frac{1}{\zeta^2\rho}\left(\frac{a_2}{\alpha_{RD}} + \frac{a_4}{\alpha_{SD}}\right)\right) \right] - \frac{e^\eta}{2 \ln 2} \text{Ei}(-\eta). \quad (25)$$

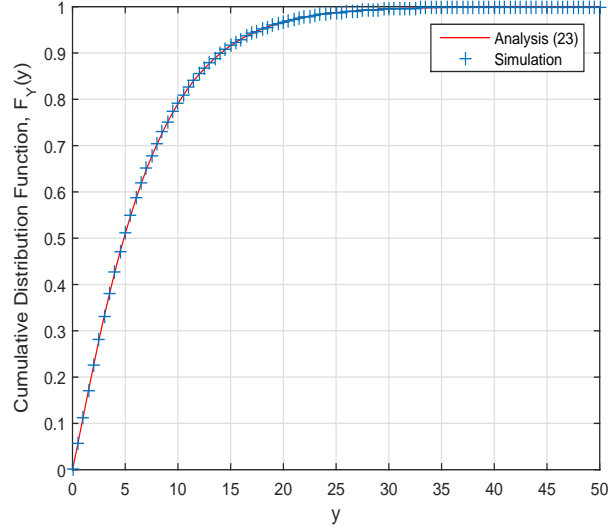


Fig. 2. Comparison between the analytical results for the CDF (23) and the Monte Carlo simulation.

IV. POWER ALLOCATIONS

This section presents appropriate power allocations of the source and the relay for our proposed scheme. Using the approximations of $\text{Ei}(-x) = Ec + \ln(x)$ and $e^x = 1 + x$, for small x , where Ec denotes the Euler constant, the ergodic SR in (25) can be approximately expressed as

$$\begin{aligned} \tilde{C}_{sum} &\sim \frac{1}{2} \log_2 \left(\frac{a_1}{a_2} \right) \left(1 + \frac{1}{\zeta^2 \rho} \left(\frac{a_2}{\alpha_{RD}} + \frac{a_4}{\alpha_{SD}} \right) \right) - \frac{1 + \left(\frac{a_1 \alpha_{SD} + a_3 \alpha_{RD}}{\zeta^2 \alpha_{SD} \alpha_{RD}} + \frac{1}{a_2 \alpha_{SR}} \right) \frac{1}{\rho}}{2 \ln 2} \left(Ec + \ln \left(\left(\frac{a_1 \alpha_{SD} + a_3 \alpha_{RD}}{\zeta^2 \alpha_{SD} \alpha_{RD}} + \frac{1}{a_2 \alpha_{SR}} \right) \frac{1}{\rho} \right) \right) \\ &\sim \frac{1}{2} \log_2 \left(\frac{a_1 \zeta^2 \alpha_{SD} \alpha_{RD} \alpha_{SR}}{a_2 \alpha_{SR} (a_1 \alpha_{SD} + a_3 \alpha_{RD}) + \zeta^2 \alpha_{SD} \alpha_{RD}} \right) - \frac{Ec}{2 \ln 2} + \frac{1}{2} \log_2 \rho. \end{aligned} \quad (26)$$

Letting the derivative of (26) with respect to the power allocation factors a_1 and a_3 be 0, i.e., $\frac{\partial \tilde{R}_{sum}}{\partial a_i} = 0$, for $i = 1, 3$, and assuming $\varphi_1 = \sqrt{\frac{1-a_3}{a_1}} - \sqrt{\frac{a_3}{1-a_1}}$, $\varphi_2 = \sqrt{\frac{a_1}{1-a_3}} - \sqrt{\frac{1-a_1}{a_3}}$, and $\Psi = \sqrt{a_1(1-a_3)} + \sqrt{(1-a_1)a_3}$, the optimal power allocation factor a_1 and a_2 , for $0 < a_i < 1$, can be obtained from (27) and (28).

$$\frac{a_1 \Psi (\varphi_1 \Psi \alpha_{RD} \alpha_{SD} + \alpha_{SR} \alpha_{SD} - a_3 \alpha_{SR} \alpha_{RD} - 2a_1 \alpha_{SD} \alpha_{SR})}{\Psi^2 \alpha_{RD} \alpha_{SD} + (1-a_1)(a_3 \alpha_{RD} + a_1 \alpha_{SD}) \alpha_{SR}} - a_1 \varphi_1 - \Psi = 0, \quad (27)$$

$$\frac{a_1 \Psi (\varphi_2 \Psi \alpha_{RD} \alpha_{SD} + (1-a_1) \alpha_{RD} \alpha_{SR})}{\Psi^2 \alpha_{RD} \alpha_{SD} + (1-a_1)(a_3 \alpha_{RD} + a_1 \alpha_{SD}) \alpha_{SR}} - a_1 \varphi_2 = 0. \quad (28)$$

V. NUMERICAL RESULTS

In this section, we examine the performance of our proposed two-stage power allocation NOMA for CRN in terms of the ergodic SR with fixed $\alpha_{SD} = 1$. All results are averaged over 20,000 channel realizations. Comparisons are made with the CRS-NOMA [4] in two considered system setups: (1) $\alpha_{SR} = 10$, $\alpha_{RD} = 2$; (2): $\alpha_{SR} = 2$, $\alpha_{RD} = 10$.

Fig. 3 depicts the ergodic SR performance versus the transmit SNR with fixed $a_1 = 0.95$ and $a_3 = 0.05$. It is easy to see

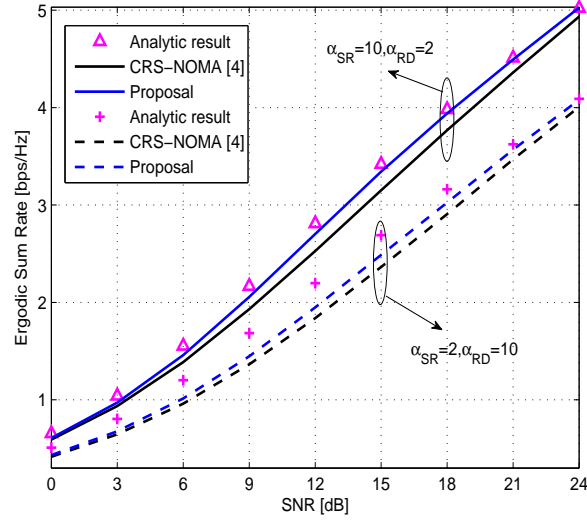


Fig. 3. The ergodic SRs achieved by our proposal and CRS-NOMA [4] versus the transmit SNR.

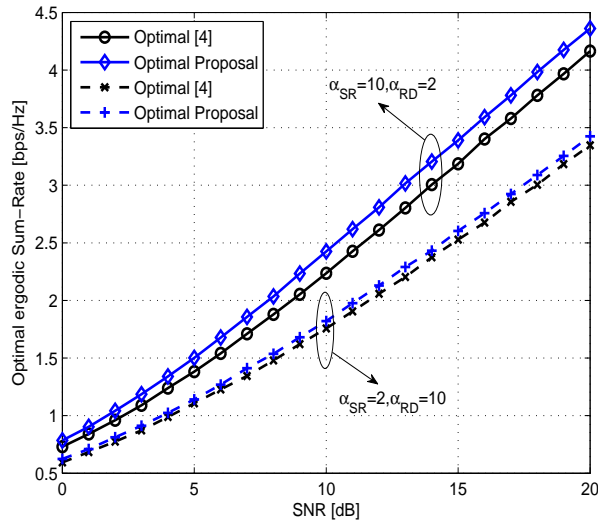


Fig. 4. The ergodic SRs achieved by our proposed scheme and CRS-NOMA [4] with the optimal allocation schemes versus the transmit SNR.

that there is a good match between the analytical result in (25) and the simulation result. Furthermore, the ergodic SR of our proposal outperforms the one in [4]. This is because the achievable rate is dominated by the receive SNR at the destination which is always smaller than C_{x_1} . In addition, by employing the two-stage power allocation and the MRC, the SNR gains can be further improved. Remarkably, there is a gap between the simulation results and the analytic results in for the small α_{SR} , this is because that the approximations of $\frac{\alpha_{SR} a_1 \rho}{\alpha_{SR} a_1 \rho}$

By using the exhaustive search to obtain the optimal power allocation factor $\{a_1, a_3\}$ and (26)-(28) to obtain the suboptimal one, in Fig. 4 and Fig. 5 we compare the optimal and sub optimal ergodic SR for our proposed scheme versus the transmit SNR with the optimal one in [4]. Fig. 4 shows that the optimal ergodic SR of our proposal overwhelms the optimal one for

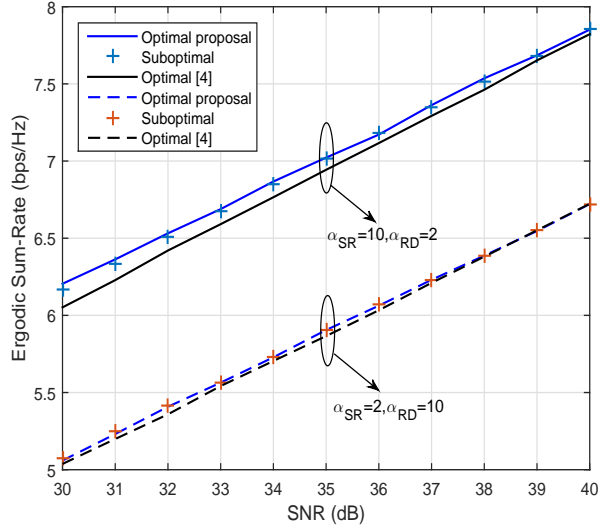


Fig. 5. The ergodic SRs achieved by our proposed scheme and CRS-NOMA [4] with the optimal and the suboptimal power allocation schemes versus the high transmit SNR.

CRS-NOMA in the 0 ~ 20 dB SNR region. In Fig. 5, it easy to see that the suboptimal solution is close to the optimal one at high SNR, which supports the practical utility of our design. Remarkably, in the typically high SNR region, the performance of our proposed scheme and CRS-NOMA are close. This observation is consistent with (26) and [4, Eq. (15)], where these equations are resulted as $\frac{1}{2}\log_2\rho$ at high SNR. In addition, for both Fig. 4 and Fig. 5, the advantage is the greatest for the case $\alpha_{SR} > \alpha_{RD}$.

VI. CONCLUSIONS

In this paper, we have proposed a two-stage power allocation CRS using NOMA and derived the closed-form solution for the proposed system. By means of simulation results, it has been shown that our proposed scheme significantly improves the ergodic SR compared with the CRS-NOMA. In addition, a suboptimal solution obtained by the practical power allocation scheme has been also provided, which is close to the optimal one at high SNR.

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